

Principles of Communication Systems

Module - 2

Angle Modulation

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Angle Modulation:

It is a process in which either frequency or phase [i.e angle] of the carrier is varied in accordance with the instantaneous amplitude of the message signal, keeping amplitude of the carrier wave constant.

Angle modulated wave can be expressed as,

$$s(t) = A_c \cos[\theta(t)]$$

where,

A_c = carrier amplitude, maintained constant

$\theta(t)$ = angular argument which is varied in proportion with the message signal $m(t)$.

Depending on the $\theta(t)$ changes, there are two types of angle modulation.

1) Frequency Modulation

2) Phase Modulation.

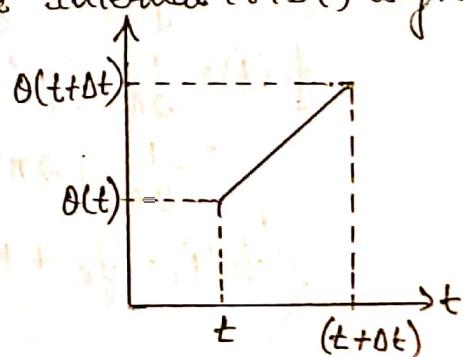
Instantaneous frequency of angle modulated wave $f_i(t)$:

* The variation of $\theta(t)$ due to $m(t)$ can be expressed mathematically based on the type of angle modulation.

* $\theta(t)$ changes by 2π radians to complete one oscillation, if $\theta(t)$ is increased monotonically with time as shown in figure:

* Then the average frequency over the interval $(t + \Delta t)$ is given by

$$f_{avg} = f_{\Delta t}(t) = \frac{\theta(t + \Delta t) - \theta(t)}{\Delta t}$$



Then the instantaneous frequency of the angle modulated wave $s(t)$ is given by,

$$\begin{aligned} f_i(t) &= \lim_{\Delta t \rightarrow 0} \omega(t) \\ &= \lim_{\Delta t \rightarrow 0} \left[\frac{\theta(t+\Delta t) - \theta(t)}{2\pi \Delta t} \right] \\ f_i(t) &= \frac{1}{2\pi} \frac{d\theta(t)}{dt} \end{aligned}$$

Note: For an unmodulated carrier, the angle $\theta(t)$ is
 $\theta(t) = 2\pi f_c t + \phi$

Phase modulation:

PM is defined as the form of angle-modulation in which the angular argument ' $\theta(t)$ ' is varied linearly with the message signal ' $m(t)$ ' as given below:

$$\text{i.e } \theta(t) = 2\pi f_c t + k_p m(t)$$

Where, $k_p \rightarrow$ phase sensitivity of the modulator, which is constant.

w.k.t $S_{\text{angle}}(t) = A_c \cos [\theta(t)]$
modulated

Then, the phase modulated wave $S_{\text{pm}}(t)$ is given by,

$$S_{\text{pm}}(t) = A_c \cos [2\pi f_c t + k_p m(t)]$$

Instantaneous frequency $f_i(t)$ in PM wave:

w.k.t $f_i(t) = \frac{1}{2\pi} \frac{d\theta(t)}{dt}$

In PM, $\theta(t) = 2\pi f_c t + k_p m(t)$

Substituting in $f_i(t)$, we get

$$f_i(t) = \frac{1}{2\pi} \frac{d}{dt} [2\pi f_c t + k_p m(t)]$$

$$= \frac{1}{2\pi} [2\pi f_c + k_p \frac{d}{dt} m(t)]$$

$$\left\langle f_i(t) = f_c + k_p \frac{1}{2\pi} \frac{dm(t)}{dt} \right\rangle$$

Thus in phase modulation, instantaneous frequency $f(t)$ varies linearly with the derivative of $m(t)$.

Frequency Modulation:

It is the form of angle modulation in which the instantaneous frequency $f(t)$ is varied linearly with the message signal $m(t)$.

$$\text{i.e } \langle f(t) = f_c + k_f m(t) \rangle$$

where,

k_f = Frequency sensitivity, which is constant.

Angular argument in FM [$\theta(t)$]:

$$\text{W.K.T } f_i(t) = \frac{1}{2\pi} \frac{d\theta(t)}{dt}$$

$$d\theta(t) = 2\pi f_i(t) dt$$

Integrate on B.S w.r.t t

$$\int d\theta(t) = \int^t 2\pi f_i(t) dt$$

$$\theta(t) = 2\pi \int^t f_i(t) dt$$

Substituting $f_i(t) = f_c + k_f m(t)$, we get

$$\text{stationary } \theta(t) = 2\pi \int^t [f_c + k_f m(t)] dt$$

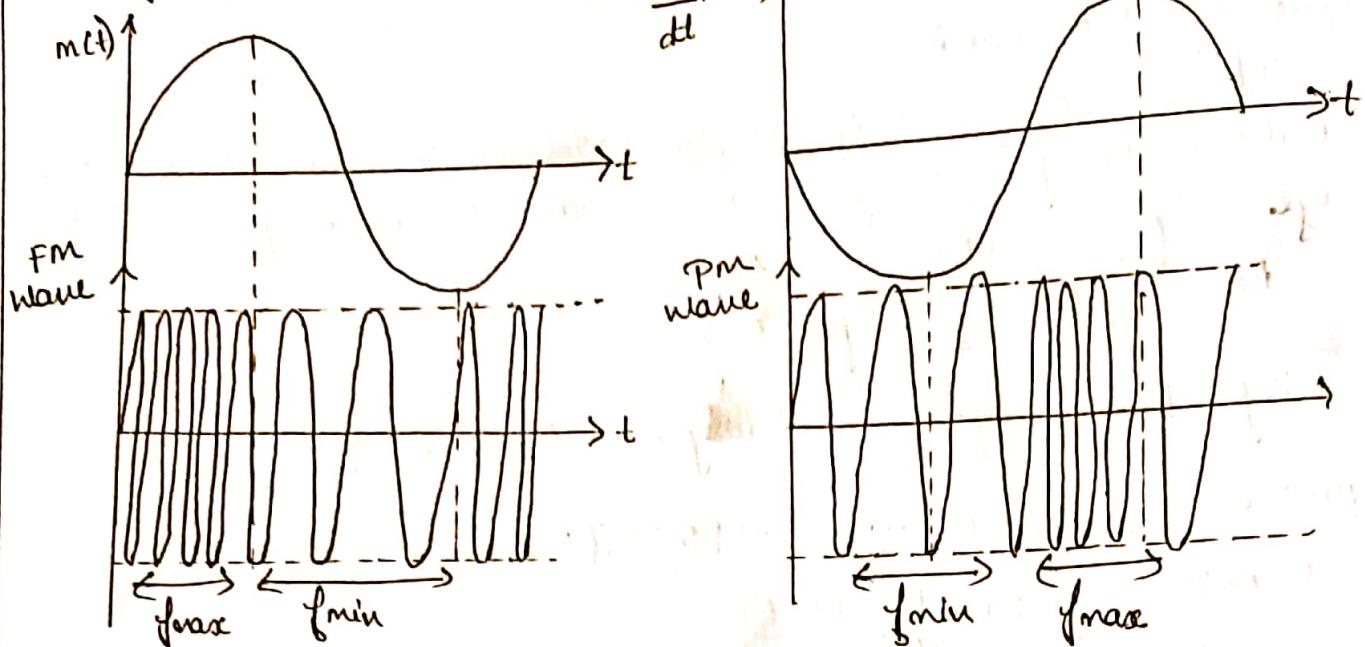
$$\theta(t) = 2\pi f_c t + 2\pi k_f \int^t m(t) dt$$

Thus, frequency modulated wave $s_{FM}(t)$ is given by,

$$\langle s_{FM}(t) = A_c \cos [2\pi f_c t + 2\pi k_f \int_0^t m(t) dt] \rangle$$

Thus in frequency modulation, angular argument is varied linearly with the integral of the $m(t)$.

Waveforms:



Relationship between FM and PM

- Generation of
 - i) FM using PM
 - ii) PM using FM.

The time domain expression for FM and PM wave are

$$FM \text{ wave: } S_{FM}(t) = A_c \cos [2\pi f_c t + 2\pi k_f \int_0^t m(t) dt]$$

$$PM \text{ wave: } S_{PM}(t) = A_c \cos [2\pi f_c t + k_p m(t)]$$

Comparing above two expressions, we can conclude that FM wave is same as PM wave, if $m(t)$ in PM wave is replaced by $\int_0^t m(t) dt$.

i) Generation of FM wave using PM [Phase modulator]

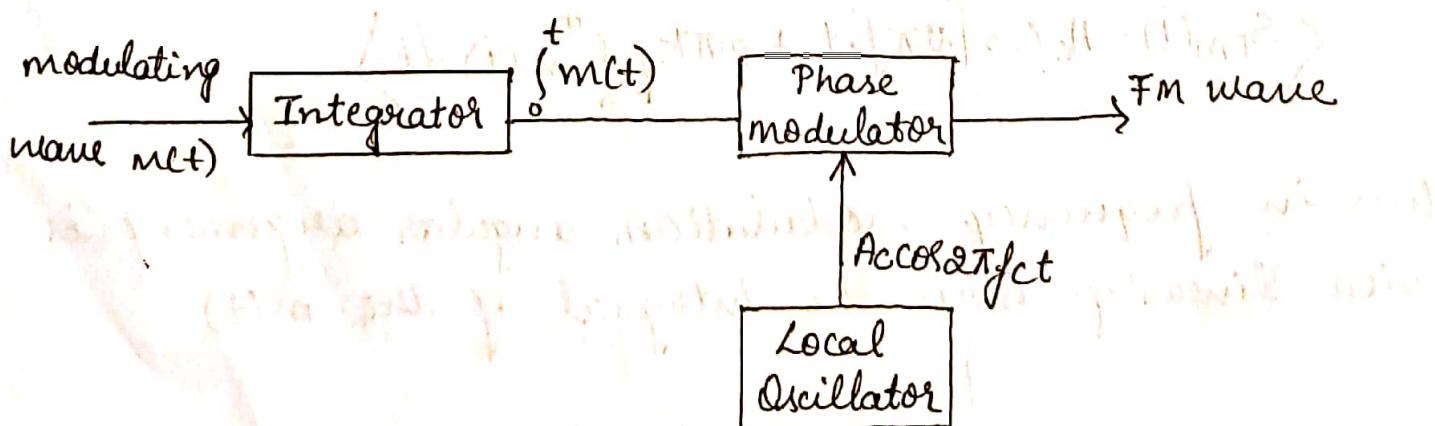


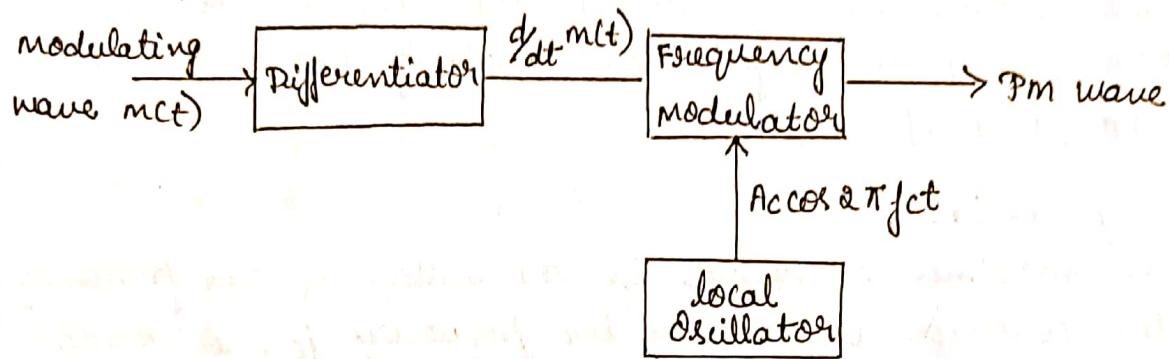
Fig: Generation of FM from phase modulator

FM wave can be generated by first integrating $m(t)$ and then it is used as input for phase modulator to produce FM wave at the output as shown in figure.

$$\langle S(t) = A_c \cos [2\pi f_c t + 2\pi k_f \int_0^t m(t) dt] \rangle$$

\downarrow
 K_p

ii) Generation of PM wave using FM [Frequency modulator]
using frequency modulator, PM is generated by first differentiating modulating signal $m(t)$ and then input to the frequency modulator as shown in below figure



w.k.t $S_{FM}(t) = A_c \cos [2\pi f_c t + 2\pi k_f \int_0^t m(t) dt]$

Input to frequency modulator is $\frac{d}{dt} m(t)$

Thus $= A_c \cos [2\pi f_c t + 2\pi k_f \frac{d}{dt} \int_0^t m(t) dt]$

~~$= A_c \cos [2\pi f_c t + 2\pi k_f m(t)]$~~

Substituting $2\pi k_f = K_p$

$$\langle S_{PM}(t) = A_c \cos [2\pi f_c t + K_p m(t)] \rangle$$

Single tone Frequency Modulation:

w.k.t $S_{FM}(t) = A_c \cos [2\pi f_c t + 2\pi k_f \int_0^t m(t) dt]$

Since single tone, consider $m(t) = A_m \cos 2\pi f_m t$

$$S_{FM}(t) = A_c \cos [2\pi f_c t + 2\pi k_f \int_0^t A_m \cos 2\pi f_m t dt]$$

$$= A_c \cos [2\pi f_c t + 2\pi k_f A_m \int_0^t \cos 2\pi f_m t dt]$$

$$= A_c \cos [2\pi f_c t + 2\pi k_f A_m \frac{\sin 2\pi f_m t}{2\pi f_m}]$$

$$S_{fm}(t) = A_c \cos [2\pi f_c t + \frac{\Delta f}{f_m} \sin 2\pi f_m t]$$

$$\therefore k_f A_m = \Delta f$$

$$S_{fm}(t) = A_c \cos [2\pi f_c t + \beta \sin 2\pi f_m t]$$

$$\therefore \beta = \Delta f / f_m$$

where, Δf = frequency deviation
 β = modulation index.

Modulation index:

It is the ratio of frequency deviation Δf to the modulating frequency ' f_m '. $[\beta = \Delta f / f_m]$

In FM, the modulation index is very important, because it decides the bandwidth of the FM wave.

If modulation index is small, then FM is Narrow Band FM [NBFM], if β is large then resulting FM is wide Band FM [WBFM].

Frequency Deviation:

The maximum change in the instantaneous frequency from the average value carrier frequency f_c , is known as frequency deviation.

$$\langle \Delta f = k_f A_m \rangle$$

Transmission Bandwidth:

The FM wave consists infinite number of sidebands thus Bandwidth of a FM signal is infinite.

In practical, By Carson's rule,

$$BW = 2(\Delta f + f_m)$$

Other forms

$$BW = 2\Delta f \left(1 + \frac{f_m}{\Delta f}\right)$$

$$BW = 2f_m \left(\frac{\Delta f}{f_m} + 1\right)$$

$$BW = 2\Delta f \left(1 + \frac{1}{\beta}\right)$$

$$BW = 2f_m (\beta + 1)$$

$$\therefore \beta = \Delta f / f_m$$

Types of FM or classification of FM

Depending on the value of the modulation index ' β ', FM wave is classified as follows,

- 1) Narrow-band FM (NBFM)
- 2) Wideband FM (WBFM)

Narrow-Band FM [NBFM]:

If the modulation index is small value, i.e less than one radian then it is Narrow Band FM.

Generation of NBFM with block diagram and its spectrum

W.K.T Angle modulated wave is given by,

$$S_{\text{angle}}(t) = A_c \cos[\omega \pi f c t + \phi] \quad \rightarrow ①$$

modulated wave

$$\text{using } \cos(A+B) = \cos A \cdot \cos B - \sin A \cdot \sin B$$

$$\therefore A = \omega \pi f c t, B = \phi$$

$$S_{\text{angle}}(t) = A_c [\cos \omega \pi f c t \cos \phi - \sin \omega \pi f c t \sin \phi] \quad \rightarrow ②$$

modulated wave

In NBFM, β is small & ϕ is less than 1 radian, hence it is possible to approximate

$$\cos \phi \approx 1 \quad \rightarrow ③$$

$$\sin \phi \approx \phi$$

substitute Eq (3) in Eq (2), we get

$$S(t) = A_c [\cos \omega \pi f c t - \sin \omega \pi f c t \phi]$$

NBFM

$$[W.K.T] = S_{\text{FM}}(t) = A_c \cos[\omega \pi f c t + \omega \pi k_f \int_0^t m(t) dt] \quad \rightarrow ④$$

Compare Eq (4) and Eq (1),

$$\phi \text{ in FM} = \omega \pi k_f \int_0^t m(t) dt$$

By substituting ϕ in $S_{\text{NBFM}}(t)$, we get

$$S_{\text{NBFM}}(t) = A_c \cos \omega \pi f c t - A_c \sin \omega \pi f c t \phi$$

$$\langle S_{\text{NBFM}}(t) = A_c \cos \omega \pi f c t - A_c \sin \omega \pi f c t \cdot \omega \pi k_f \int_0^t m(t) dt \rangle$$

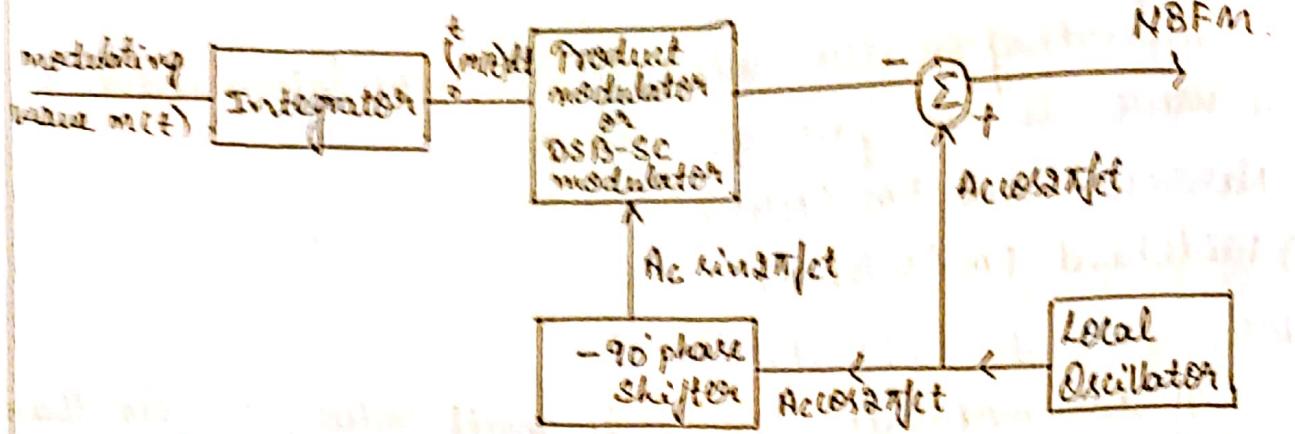


fig: Narrow Band phase modulator

Spectrum of NBPM:
we get,

$$S_{NBPM}(t) = Ac \cos \pi fct - \omega \pi k_f A_c \sin \pi fct \int^t m(t) dt$$

$$\text{take } m(t) = A_m \cos 2\pi f_m t$$

$$= Ac \cos 2\pi fct - \omega \pi k_f A_c \sin \pi fct \left(A_m \cos 2\pi f_m t \right) dt$$

$$= Ac \cos 2\pi fct - \frac{\omega \pi k_f A_m A_c}{2} \sin \pi fct \cdot \sin 2\pi f_m t$$

$$= Ac \cos 2\pi fct - \frac{k_f A_m}{2} A_c \sin \pi fct \sin 2\pi f_m t$$

$$\therefore \Delta f = \frac{k_f A_m}{2} \quad \beta = \frac{\Delta f}{f_m}$$

$$= Ac \cos 2\pi fct - \frac{\Delta f}{f_m} A_c \sin \pi fct \sin 2\pi f_m t$$

$$= Ac \cos 2\pi fct - \beta A_c \sin \pi fct \sin 2\pi f_m t$$

using $\sin A \cdot \sin B = -\frac{1}{2} [\cos(A+B) - \cos(A-B)]$

$$S_{NBPM}(t) = Ac \cos 2\pi fct + \frac{\beta A_c}{2} [\cos \pi(f_c + f_m)t - \cos \pi(f_c - f_m)t]$$

$$(S(t))_{NBPM} = Ac \cos 2\pi fct + \frac{\beta A_c}{2} \cos \pi(f_c + f_m)t - \frac{\beta A_c}{2} \cos \pi(f_c - f_m)t$$

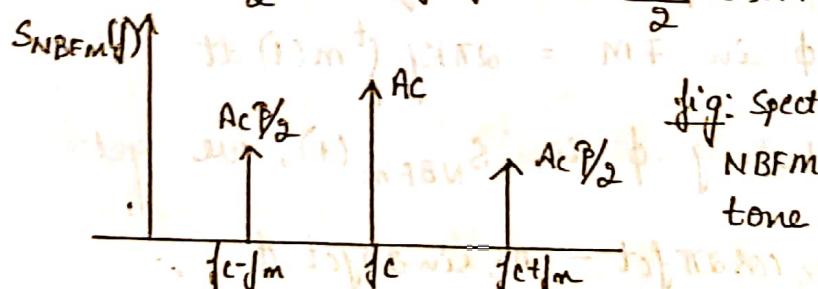


fig: Spectrum content of NBPM for single tone modulation

- Note:
- * NBPM spectrum is same as AM conventional, it contains carrier signal, upper sideband (USB) & Lower Sideband (LSB).
 - * Transmission BW of NBPM is $2\pi f_m$

Wideband FM:

If the modulation index is large value, then it is wideband FM.

Time domain expression for wideband or expression for the spectrum of FM wave:

consider single tone frequency modulation expression,

$$\text{S}_{\text{STFM}}(t) = A_c \cos[\alpha \pi f_c t + B \sin \alpha \pi f_m t]$$

$$\text{using } \cos(A+B) = \cos A \cos B - \sin A \sin B$$

$$\text{let } A = \alpha \pi f_c t, B = B \sin \alpha \pi f_m t$$

$$\text{S}_{\text{STFM}}(t) = A_c [\cos \alpha \pi f_c t \cdot \cos(B \sin \alpha \pi f_m t) - \sin \alpha \pi f_c t \cdot \sin(B \sin \alpha \pi f_m t)]$$

$$\text{W.K.T } s(t) = S_I(t) \cos \alpha \pi f_c t - S_Q(t) \sin \alpha \pi f_c t$$

$$\text{from above eq } s_I(t) = \cos(B \sin \alpha \pi f_m t)$$

$$s_Q(t) = \sin(B \sin \alpha \pi f_m t)$$

from complex envelope, we have $\langle \tilde{s}(t) = s_I(t) + j s_Q(t) \rangle$

$$\tilde{s}(t) = A_c [\cos(B \sin \alpha \pi f_m t) + j \sin(B \sin \alpha \pi f_m t)]$$

$$\text{W.K.T} = e^{j\theta} = \cos \theta + j \sin \theta$$

$$\tilde{s}(t) = A_c e^{j(B \sin \alpha \pi f_m t)} \quad \because \theta = B \sin \alpha \pi f_m t.$$

Expanding using fourier series, in terms of Bessel function $J_n(\beta)$, $\tilde{s}(t)$ becomes

$$\tilde{s}(t) = A_c \sum_{n=-\infty}^{\infty} J_n(\beta) e^{j n \alpha \pi f_m t}$$

Time domain expression for WBFM is

$$\begin{aligned} S_{\text{WBFM}}(t) &= \text{Re} [\tilde{s}(t) e^{j \alpha \pi f_c t}] \\ &= \text{Re} [A_c \sum_{n=-\infty}^{\infty} J_n(\beta) e^{j n \alpha \pi f_m t} e^{j \alpha \pi f_c t}] \\ &= \text{Re} [A_c \sum_{n=-\infty}^{\infty} J_n(\beta) e^{j \alpha \pi (f_c + n f_m) t}] \end{aligned}$$

$$S_{\text{WBFM}}(t) = \text{Re} [A_c \sum_{n=-\infty}^{\infty} J_n(\beta) [\cos \alpha \pi (f_c + n f_m) t + j \sin \alpha \pi (f_c + n f_m) t]]$$

taking only the real terms, we get

$$S_{WBFM}(t) = A_c \sum_{n=-\infty}^{\infty} J_n(\beta) \cos 2\pi (fc + \eta/m)t$$

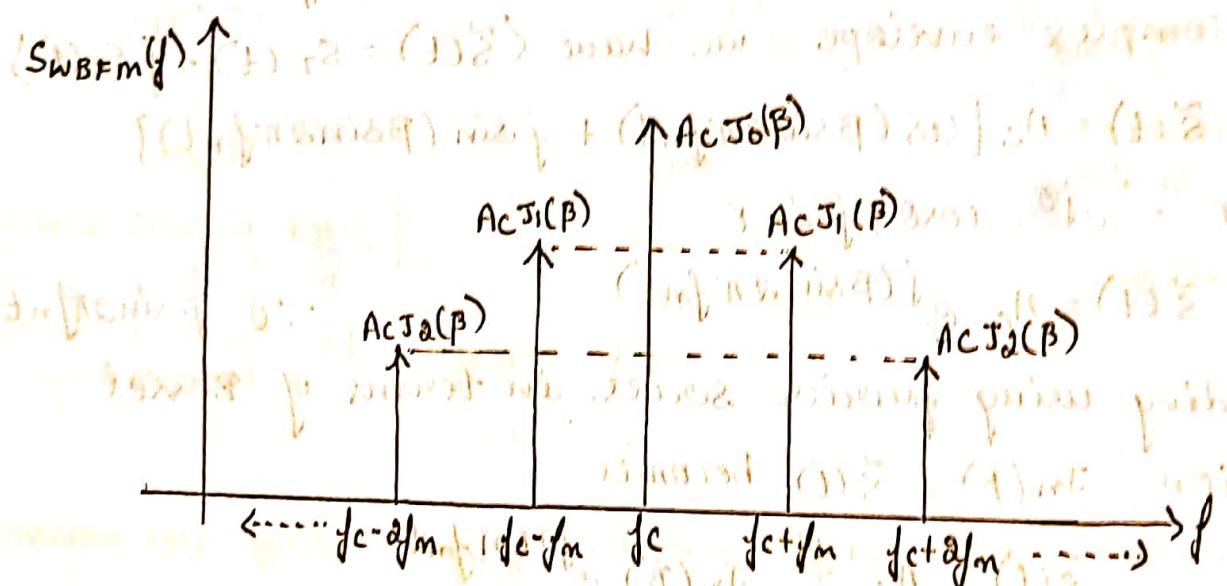
n lies between $-n, n$

i.e., $n = 0, \pm 1, \pm 2, \dots, \pm m, -n$

$$S_{WBFM}(t) = A_c [J_0(\beta) \cos 2\pi fct + J_1(\beta) \cos 2\pi (fc + \eta/m)t + J_{-1}(\beta) \cos 2\pi (fc - \eta/m)t + J_2(\beta) \cos 2\pi (fc + 2\eta/m)t + \dots]$$

Note:

$$J_n(\beta) = \begin{cases} J_n(\beta) & \text{for } n \text{ even} \\ -J_n(\beta) & \text{for } n \text{ odd} \end{cases}$$



Spectrum of WBFM

Thus WBFM consists of carrier signal and infinite number of sidebands.

Differences between NBFM and WBFM

Parameters	NBFM	WBFM
* Modulation Index(β)	$\beta \leq 1$	$\beta > 1$
* Spectrum	The spectrum of NBFM is same as that of AM. Contains 2 sidebands & carrier.	The spectrum of WBFM differs from AM. Contains carrier & infinite number of sidebands.
* Bandwidth	small	large
* Noise Suppression	poor	better
* Range of modulating frequency	30Hz to 3KHz	30Hz to 15KHz
* Maximum Deviation [Δf]max	5KHz	$\pm 5KHz$
* Transmission quality	low	high
* Applications	used in speech transmission. Ex: FM mobile comm.	used for high quality music transmission. Ex: Entertainment broadcasting.

Generation of FM waves:

There are two basic methods of generating FM waves, namely,

1) Indirect method or Indirect FM:

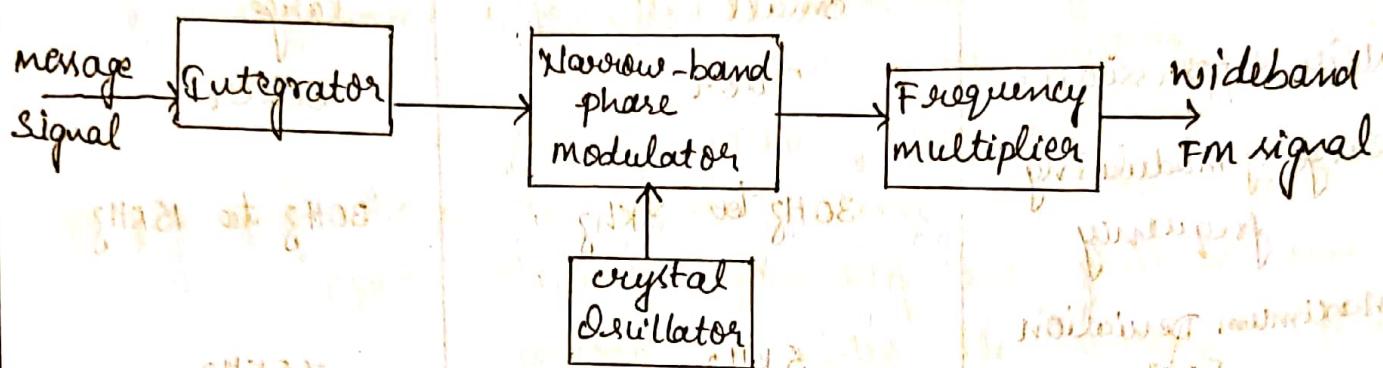
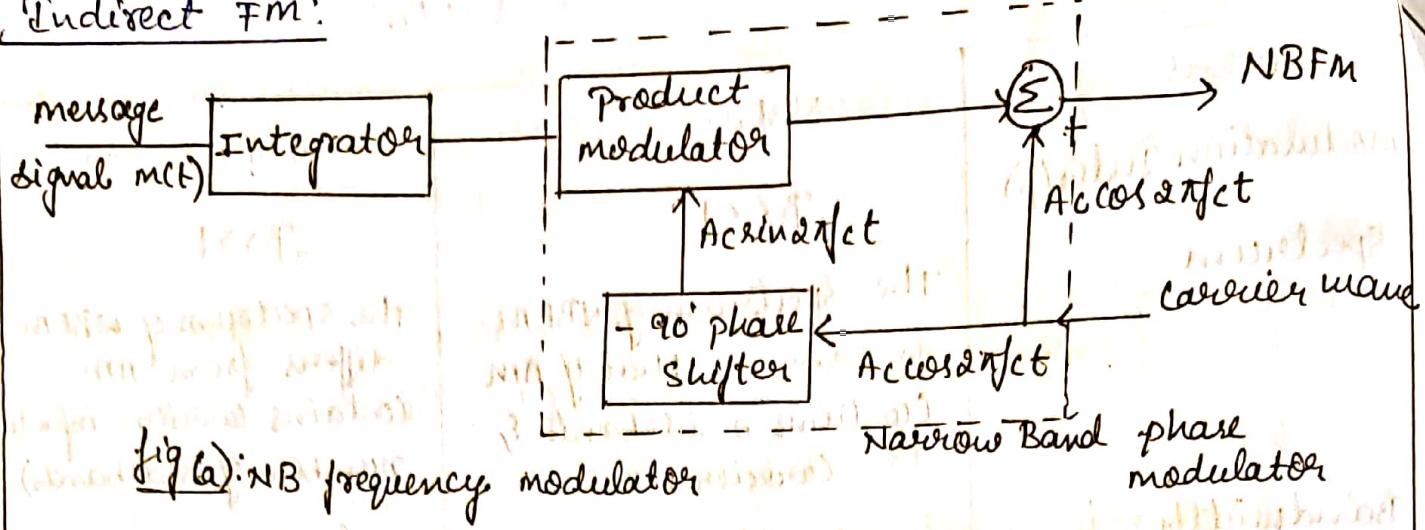
In this method, a NBFM wave is generated, frequency multipliers are then used to increase the frequency deviation which results in wideband - FM (WBFM).

2) Direct FM or Direct method:

In direct FM, the carrier frequency f_c is directly varied in accordance with the amplitude of the modulating signal.

Direct FM is not feasible, practically as it involves maintaining high frequency stability of the carrier with adequate frequency deviation.

Indirect FM:

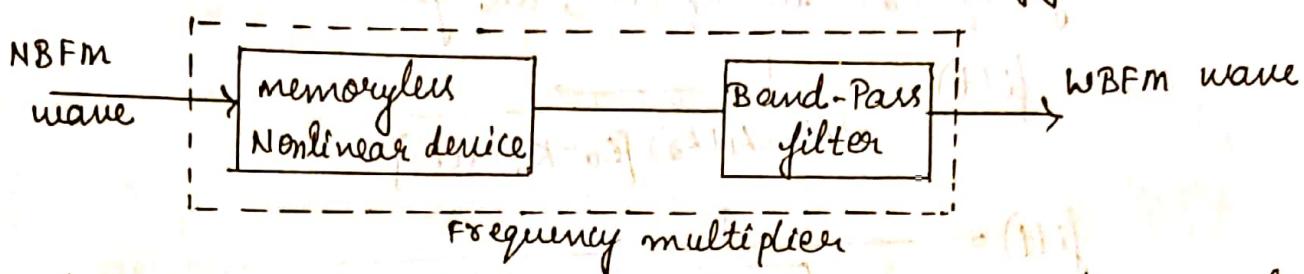


fig(b): wide-band frequency modulator.

- * In indirect method, the message signal $m(t)$ is first passed through an integrator before applying it to the phase modulator as shown in fig(a).
- * The carrier signal is generated by using crystal oscillator because it provides very high frequency stability.
- * The operation of indirect method is divided into two parts as follows:
 - Generate a NBFM using a phase modulator
 - using the frequency multipliers & mixer to obtain the required values of frequency deviation and modulation index (i.e WBFM).
- * In order to minimize the distortion in the phase modulator, the maximum phase deviation or modulation index ' β ' is kept small thereby resulting in a NBFM signal.

Generation of WBFM:

* The output of the narrow band phase modulator is then multiplied by a frequency multiplier, providing the desired WBFM wave as shown in below figure.



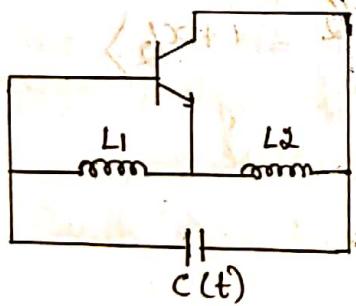
* A frequency multiplier consists of a memoryless non-linear device followed by a BPF.

The BPF has two functions to perform:

- To pass the FM wave centered at carrier frequency ω_1 and having the frequency deviation $\Delta\omega_1$.
- To suppress all other FM spectra.

* The output of the frequency multiplier produces the desired wide-band FM wave.

Direct method for FM Generation:



In the direct method of FM generation, the instantaneous frequency of the carrier wave is varied directly in accordance with the message signal by means of a device known as voltage-controlled oscillator (VCO).

As an example, consider Hartley oscillator as shown in fig. The capacitive component of the frequency determining network consists of a fixed capacitor shunted by a voltage variable capacitor, commonly known as varactor or varicap. Thus the resultant capacitance is $C(t)$.

The frequency of oscillation of the Hartley oscillator is given by

$$f_i(t) = \frac{1}{2\pi\sqrt{(L_1 + L_2)C(t)}} \quad \rightarrow ①$$

where, $C(t)$ is total capacitance of fixed & variable voltage capacitor. L_1 & L_2 are two inductances in frequency determining network.

$$C(t) = C_0 - k_C m(t) \rightarrow ②$$

where, $C_0 \rightarrow$ capacitance value in the absence of modulation.
 $k_C \rightarrow$ variable capacitor's sensitivity.

Substituting Eqn(2) in Eqn(1), we get

$$f_i(t) = \frac{1}{2\pi\sqrt{(L_1+L_2)}[C_0 - k_C m(t)]}$$

$$f_i(t) = \frac{1}{2\pi\sqrt{(L_1+L_2)}C_0 \left[1 - \frac{k_C}{C_0} m(t) \right]}$$

$$f_i(t) = \frac{1}{2\pi\sqrt{(L_1+L_2)}C_0} \cdot \frac{1}{\sqrt{\left[1 - \frac{k_C}{C_0} m(t) \right]}}$$

$$f_i(t) = \frac{1}{2\pi\sqrt{(L_1+L_2)}C_0} \sqrt{\left[1 - \frac{k_C}{C_0} m(t) \right]^{\frac{1}{2}}}$$

$$f_i(t) = \frac{1}{2\pi\sqrt{(L_1+L_2)}C_0} \left[1 - \frac{k_C}{C_0} m(t) \right]^{-\frac{1}{2}}$$

$$f_i(t) = f_0 \left[1 - \frac{k_C}{C_0} m(t) \right]^{-\frac{1}{2}}$$

(Binomial theorem, $\rightarrow [1-x]^{-\frac{1}{2}} = 1 + \frac{x}{2}$)

$$f_i(t) = f_0 \left[1 + \frac{k_C}{2C_0} m(t) \right]$$

$$f_i(t) \approx f_0 + \frac{k_C f_0}{2C_0} m(t)$$

$$\langle f_i(t) \rangle = f_0 + k_f m(t)$$

where, $k_f =$ frequency sensitivity $= \frac{k_C f_0}{2C_0}$

Thus $f_i(t) = f_0 + k_f m(t)$ represents standard expression for instantaneous frequency of FM wave.

The major disadvantage is that there is no frequency stability.

Stability can be achieved by using feedback system as shown in figure,

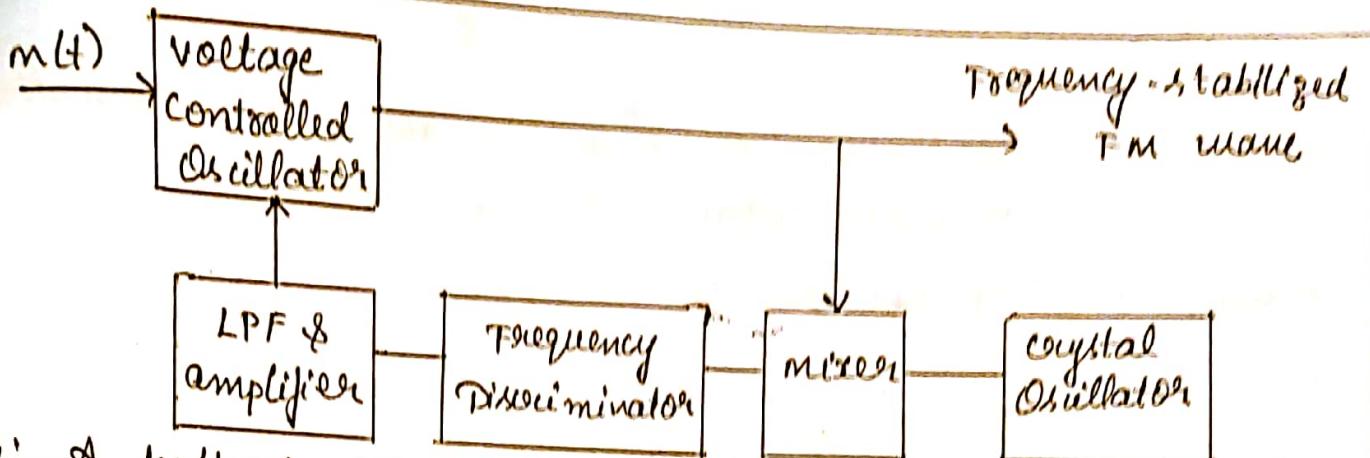


fig:- A feedback scheme for the frequency stabilization of a frequency modulator.

- * Output of the FM generator is applied to a mixer together with the output of a crystal oscillator and the difference frequency term is extracted by mixer. mixer O/P is next applied to a frequency discriminator and then low-pass filtered.
- * When the FM transmitter has exactly the correct carrier frequency, the low-pass filter output is zero. However, deviations of the transmitter carrier frequency from its assigned value will cause the frequency discriminator-filter combination to develop a dc output voltage.
- * The dc voltage, after amplification it is applied to the voltage controlled oscillator of the FM transmitter in such a way to modify the frequency of the oscillator.

De-modulation of FM waves:-

→ Frequency de-modulation is the process of recovering the original modulating wave from the frequency modulated wave.

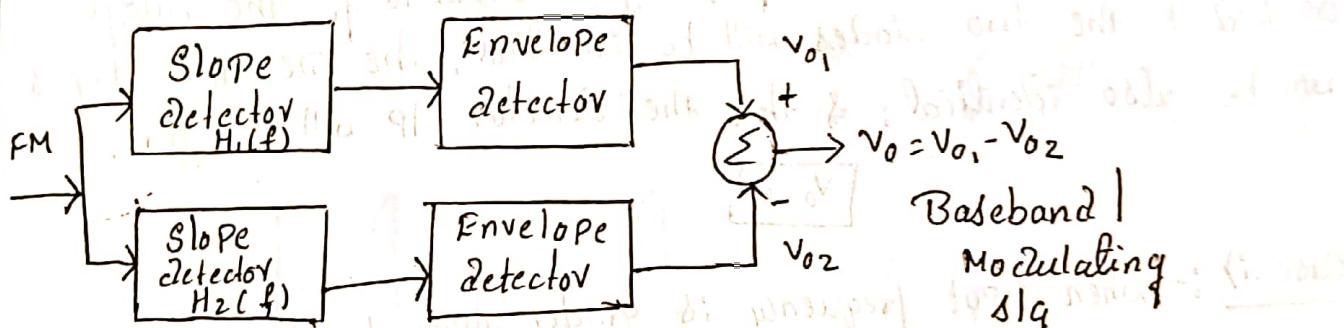
→ The FM-demodulators are classified into:-

i) Direct Method:- Eg:- Frequency discriminators, Zero crossing detectors.

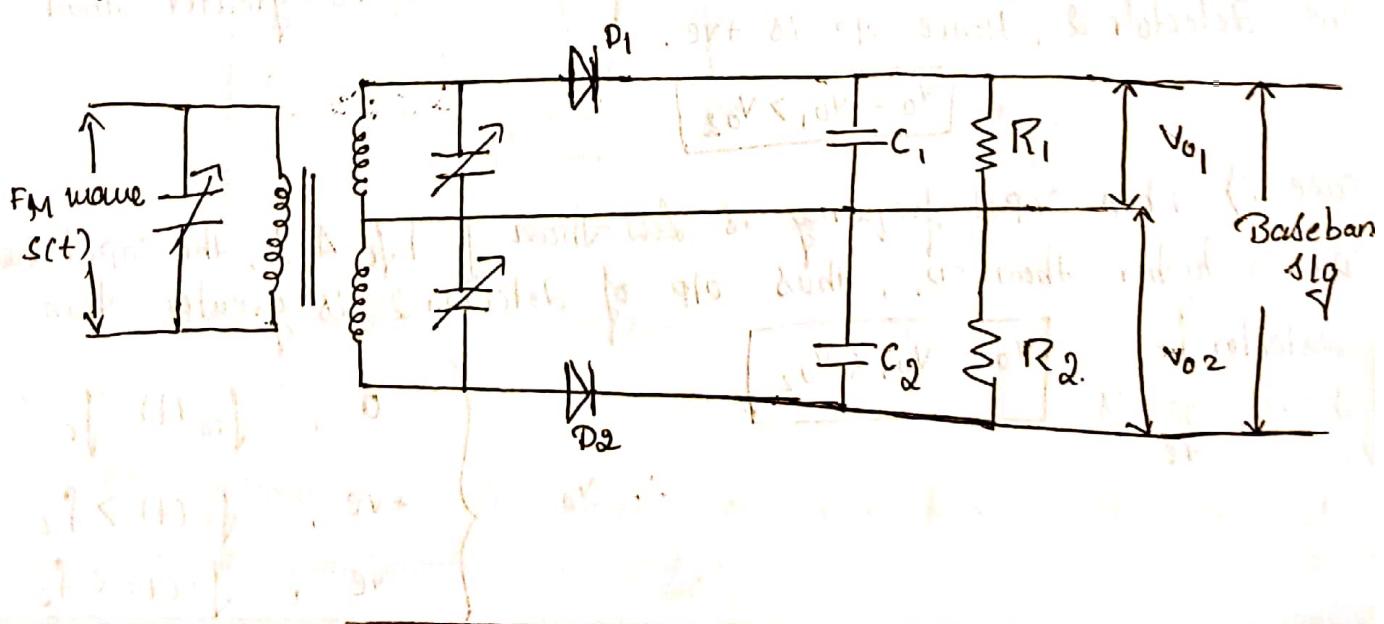
ii) Indirect Method:- Eg:- Phase-locked loop

Balanced Frequency discriminator | Balanced Slope detector :-

→ The model of the balanced discriminator as a pair of slope circuits with their complex transfer functions, followed by envelope detector & a summer as shown in the fig(a).



fig(a): Idealized model of balanced frequency discriminator



→ There are 2 tuned circuits

i) The Primary winding is tuned to frequency f_c

ii) Secondary winding is divided into 2 parts.

→ The upper tuned circuit is tuned above f_c , i.e $f_c + \Delta f$

→ The lower tuned circuit is tuned below f_c , i.e $f_c - \Delta f$

→ R_1, C_1 & R_2, C_2 are the filter circuits, V_{o1} & V_{o2} are the o/p voltage of the two slope detectors.

→ The final output voltage V_o is obtained by taking the difference of the individual o/p voltages V_{o1} & V_{o2} .

$$\text{i.e } V_o = V_{o1} - V_{o2}$$

Case i) :- When the input frequency is equal to f_c , the voltage applied to the two diodes will be identical, the DC o/p voltage will be also identical, & thus the detector o/p will be zero.

$$V_o = 0$$

Case ii) :- When input frequency is greater than f_c [$f_c + \Delta f$], the input to D_1 is higher than D_2 , thus o/p of detector-1 is greater than the detector-2, hence o/p is +ve.

$$V_o = V_{o1} > V_{o2}$$

Case iii) When input frequency is less than f_c [$f_c - \Delta f$], the input to D_2 is higher than D_1 , thus o/p of detector-2 is greater than detector-1. Hence o/p is -ve.

$$V_o = V_{o1} < V_{o2}$$

$$\therefore V_o =$$

$$\begin{cases} 0, & f_{in}(t) = f_c \\ +ve, & f_{in}(t) > f_c \\ -ve, & f_{in}(t) < f_c \end{cases}$$

Advantages :-

- This circuit is more efficient than simple slope detector
- It has better Linearity than the simple slope detector

Disadvantages :-

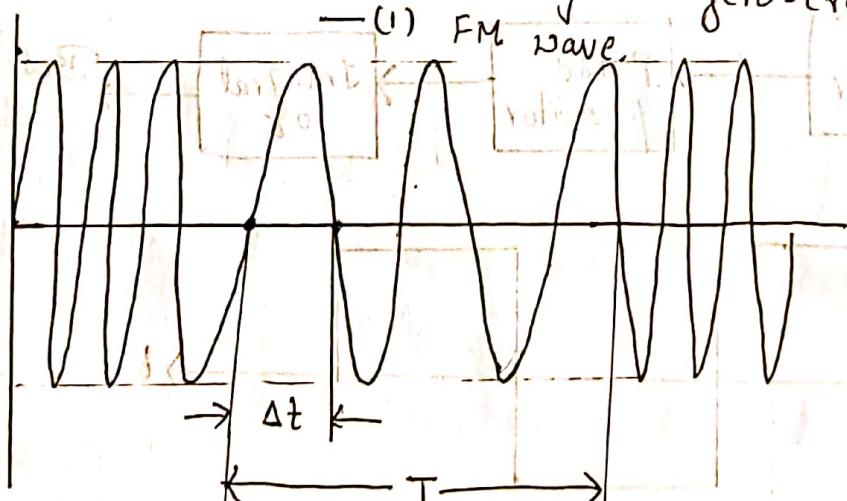
- It is difficult to tune since the 3 tuned circuits are to be tuned at different frequencies i.e. f_c , $f_c + \Delta f$, $f_c - \Delta f$.

Zero-Crossing Detector :-

- It is a frequency counter, which measures the instantaneous frequency by the number of zero-crossings. Then the rate of zero-crossings indicate the instantaneous frequency of the signal.
- Zero-crossing detector mainly operates on the principle that the instantaneous frequency of an FM wave is given by,

$$f_i \approx \frac{1}{2\Delta t}$$

Δt is the time difference b/w the adjacent zero-crossing of the



- The time interval 'T' is chosen in accordance with the following conditions.

- i) The interval 'T' is small compared to the reciprocal of the message bandwidth 'w' ($1/w$)

ii) The interval ' T ' is large compared to the reciprocal of the carrier frequency ' f_c ' of the FM wave i.e. $(1/f_c)$

→ Let n_0 denote the no: of zero-crossings inside the interval T . i.e $T = n_0 \Delta t$

$$\Delta t = T/n_0 \quad \text{(2)}$$

then, Instantaneous freqn is given by,

$$f_i = \frac{1}{2\Delta t}$$

Now, substitute Eqn (2) in (1)

$$f_i = \frac{1}{2(T/n_0)}$$

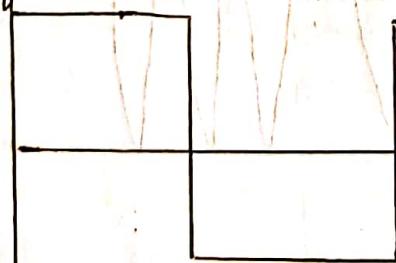
$$f_i = \frac{n_0}{2T}$$

thus $m(t)$ can be recovered by counting n_0 .



Op of multiplication

Op of Pulse generator



$\rightarrow \Delta t$

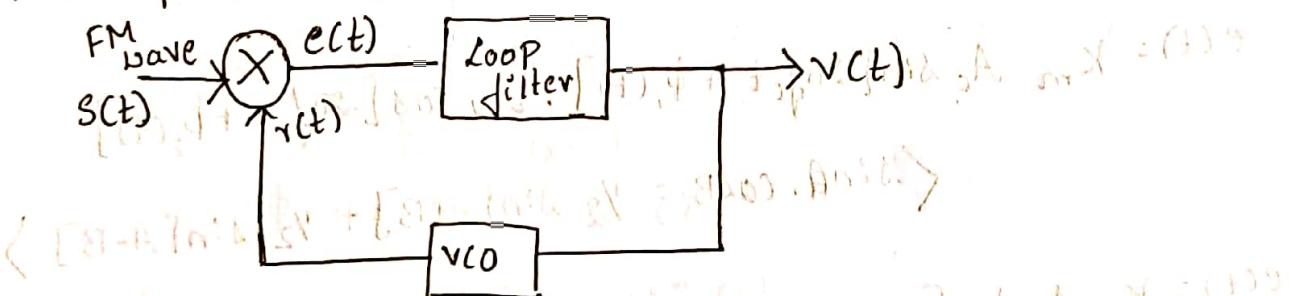


(Ex) If the width of each pulse is Δt

- Limiter Produces a square wave version of the input FM wave.
- The Pulse generator Produces a short pulses at the positive-going as well as negative-going edges of the Limiter o/p.
- Finally, integrator averages all the short pulses over an interval T , thus o/p reproduces the original modulating signal.

Phase-Locked Loop [PLL] :-

- PLL is a -ve feedback system that consists of 3-major components
 - i) A multiplier
 - ii) A Loop filter
 - iii) voltage controlled oscillator [vco], connected in the form of feedback loop as shown in the fig



- Initially assume that vco is adjusted, so that when the control voltage is zero, 2 conditions are satisfied
 - i) The frequency of the vco is precisely set at the unmodulated carrier frequency f_c .
 - ii) The vco o/p has a 90° phase-shift w.r.t un-modulated carrier wave.
- Suppose that the ip sig applied to the pll is an FM wave defined by,

$$S_{FM}(t) = A_c \sin [2\pi f_c t + 2\pi k_f \int_0^t m(t) dt] \quad (1)$$

$$S_{FM}(t) = A_c \sin [2\pi f_c t + \phi_i(t)] \quad (1)$$

$$\phi_1(t) = 2\pi k_f \int_0^t m(t).dt ; \quad k_f \rightarrow \text{freq}^n \text{ sensitivity}$$

Let the vco o/p be defined as

$$r(t) = A_v \cos [2\pi f_c t + 2\pi k_v \int_0^t v(t).dt]$$

$$r(t) = A_v \cos [2\pi f_c t + \phi_2(t)] \quad (2)$$

$$\phi_2(t) = 2\pi k_v \int_0^t v(t).dt ; \quad k_v \rightarrow \text{freq}^n \text{ sensitivity constant of the vco}$$

\rightarrow The o/p of the multiplier is,

$$e(t) = k_m s(t) \cdot r(t) \quad (3) \quad k_m = \text{multiplier gain}$$

Substitute Eqⁿ (1) & (2) in Eqⁿ (3),

$$e(t) = k_m A_c \sin [2\pi f_c t + \phi_1(t)] \cdot A_v \cos [2\pi f_c t + \phi_2(t)]$$

$$\left\langle \sin A \cdot \cos B = \frac{1}{2} \sin [A+B] + \frac{1}{2} \sin [A-B] \right\rangle$$

$$e(t) = \frac{k_m A_c A_v}{2} \left[\sin [2\pi f_c t + \phi_1(t) + 2\pi f_c t + \phi_2(t)] + \sin [2\pi f_c t + \phi_1(t) - 2\pi f_c t - \phi_2(t)] \right]$$

$$e(t) = \frac{k_m A_c A_v}{2} \left[\sin [4\pi f_c t + \phi_1(t) + \phi_2(t)] + \sin [\phi_1(t) - \phi_2(t)] \right]$$

$$e(t) = \frac{k_m A_c A_v}{2} \sin [4\pi f_c t + \phi_1(t) + \phi_2(t)] + \frac{k_m A_c A_v}{2} \sin [\phi_1(t) - \phi_2(t)] \quad (4)$$

\rightarrow The o/p of the multiplier has 2 components

i) Highest freqⁿ components, $\frac{k_m A_c A_v}{2} \sin [4\pi f_c t + \phi_1(t) + \phi_2(t)]$

ii) Lowest freqⁿ components, $\frac{k_m A_c A_v}{2} \sin [\phi_1(t) - \phi_2(t)]$

highest frequency component is eliminated by the LPF

$$e(t) = \frac{k_m A_c A_v}{2} \sin[\phi_i(t) - \phi_o(t)]$$

$$e(t) = \frac{k_m A_c A_v}{2} \sin[\phi_e(t)] \quad | \phi_e \text{ is phase error} \\ \phi_e = \phi_i(t) - \phi_o(t)$$

thus final o/p v(t) is,

$$v(t) = e(t) * h(t)$$

$$v(t) = \frac{k_m A_c A_v}{2} \sin[\phi_e(t)] * h(t) \quad | \quad (6)$$

\rightarrow To show o/p of PLL is scaled version of m(t);-

when the phase error $\phi_e(t)$ is zero, the PLL is said to be in Phase Locked.

$$\phi_e(t) = \phi_i(t) - \phi_o(t)$$

$$0 = \phi_i(t) - \phi_o(t)$$

$$\phi_i(t) = \phi_o(t) \quad | \quad (7)$$

Substitute Eqⁿ(1) & (2) in Eqⁿ(7)

$$2\pi k_f \int_0^t m(t).dt = 2\pi k_v \int_0^t v(t).dt \quad | \quad (8)$$

Differentiating above Eqⁿ(8) w.r.t to 't' on b.s

$$\frac{d}{dt} k_f \int_0^t m(t).dt = \frac{d}{dt} k_v \int_0^t v(t).dt$$

$$k_f m(t) = k_v \dot{v}(t)$$

$$v(t) = \frac{k_f}{k_v} m(t) \quad | \quad (9)$$

thus v(t), o/p of PLL is scaled version of m(t) \Leftrightarrow
 $v(t) \propto m(t)$

Comparison of Am & FM Systems:-

Parameter	Am	FM
Definition	Amplitude of the carrier signal is changed w.r.t modulating signal	Frequency of the carrier signal is changed w.r.t modulating signal
Spectrum	It has only 2 sidebands	It has 'n' no: of sidebands
Modulation Index 'B'	$M < 1$	$B > 1$
M. I formula	$M = \frac{A_m}{A_c}$	$B = \frac{\Delta f}{f_m}$
B. N	$\frac{2 f_m}{\Delta f}$	$2(\Delta f + f_m)$
Transmitted Power, P_T	$P_T = P_c \left[1 + \frac{M^2}{2} \right]$	$P_T = \frac{A_c^2}{2R}$
Application	Long dist communication	Short dist communication

→ Comparison of FM & PM Systems:-

Parameter	FM	PM
Instantaneous freq, $f_i(t)$	$f_i(t) = f_c + k_f m(t)$ $f_i(t) \propto m(t)$	$f_i(t) = f_c + k_p \frac{dm(t)}{dt}$ $f_i(t) \propto \frac{dm(t)}{dt}$
Noise Suppression	Better	Poor
SNR	Better than PM	It is inferior to that of FM
Application	FM broadcasting	Mobile systems.

- O.P of the amplifier is denoted by the letter 'fs'.
- Mixer will perform super heterodyne junction, with which will produce sum of 2 signals or differences of 2 signals.
- one of the output is from R.F Amplifier, other output is from Local oscillator which generates the carrier frequency which ranges from 1005 kHz to 2105 kHz. which is represented by f_c .
- These 2 signals are given to the mixer, the O.P of the mixer is $f_c + fs$ or $f_c - fs$. Here we are considering $f_c - fs$.
- Because, ^{only} 455kHz freqⁿ ^{Signal} should be applied to the IF-Amplifier, if we apply any frequency other than 455kHz it will reject the signal. So if we subtract $f_c - fs$ i.e $(1005\text{kHz} - 550\text{kHz}) = 455\text{kHz}$. we get 455kHz frequency sig.
- IF Amplifier will amplify the $f_c - fs$ signal, if it is given to Detector block.
- Detector will perform two operations:
 - 1) Rectification :- Negative half-cycle is eliminated, only Positive-half cycle is passed to filter.